

# Searching for dark matter via mono- $Z$ boson production at the ILC

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## Abstract

High energy colliders provide a new unique way to determine the microscopic properties of the dark matter (DM). Weakly interacting massive particles (WIMPs) are widely considered as one of the best DM candidates. It is usually assumed that the WIMP couples to the SM sector through its interactions with quarks and leptons. In this paper, we investigate the DM pair production associated with a  $Z$  boson in an effective field theory framework at the International Linear Collider (ILC), which can be used to study the interactions between the DM and leptons. For illustrative purposes, we present the integrated and differential cross sections for the  $e^+e^- \rightarrow \chi\bar{\chi}Z$  process, where the  $Z$  boson is radiated from the initial state electron or positron. Meanwhile, we analyze the neutrino pair production in association with a  $Z$  boson as the SM background.

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## I. INTRODUCTION

Observational evidence has confirmed the existence of some kind of cold non-baryonic dark matter (DM) which is the dominant component of matter in our universe [1]. However, astrophysical observations tell us nothing about the mass of the DM particle or whether it interacts with the standard model (SM) particles. In the SM, neutrinos are the only long-lived particles that interact purely via the weak force, but their masses are too small to explain the large mass component in the universe. Thus, to determine the particle nature of DM is one of the most important tasks in cosmology and particle physics.

Among the many DM candidates, weakly interacting massive particles (WIMPs) are the most compelling ones. Primarily this is due to the fact that it offers the possibility to understand the relic abundance of the DM as a natural consequence of the thermal history of the universe [2]. Theories that lie beyond the SM include various extensions of the SM, such as supersymmetry [3–6], universal extra dimensions [7] and little Higgs [8, 9], which all naturally lead to good candidates for WIMPs and the cosmological requirements for the WIMP abundance in the universe. In these theoretical frameworks, the WIMP candidates are often both theoretically well motivated and compelling. However, all of these theories still lack experimental support, and we cannot determine the new physics theory to which the DM belongs. Additionally, the first observation of the DM may come from direct or indirect detection experiments, which may not provide information about the general properties of the DM particle without offering a way to distinguish between the underlying theories. Thus, model-independent studies of DM phenomenology using effective field theory is particularly important.

Recently, the observational results favour a light DM with a mass around 10 GeV in various experiments. The DAMA experiment has reported a signal of annual modulation at a high significance level [10]. This signal is consistent with a DM discovery interpretation with a DM particle of mass  $\lesssim 10$  GeV [11, 12]. The CoGeNT and XENON10/100 collaborations have also reported a signal [13–15] which can be explained by a WIMP in this mass range. There has been much interest in light DM models (where the DM mass is order of a few GeV) [16–25]. The high energy colliders are ideal facilities for searching light WIMPs, since they are most effective when producing highly boosted light WIMPs. In the case of a WIMP, stability on the order of the lifetime of the universe implies that pair production

must highly dominate over single production, and precludes the WIMP from decaying within the detector volume. WIMPs therefore appear as missing energy, and can potentially be observed by searching for visible particles recoiling against DM particles [26–30].

The International Linear Collider (ILC) [31, 34] is a proposed positron-electron collider that is planned to operate at center of mass energy up to 500 GeV with a potential later upgrade to 1 TeV. Compared with the Large Hadron collider (LHC), the  $e^+e^-$  linear collider has a particularly clear background environment. It may have enough energy to produce WIMPs. On the other hand, positron-electron collider can play a major role in providing precision data for understanding the DM. At the ILC, the DM signal has been studied by directly detecting boson transverse energy signal, such as mono-photon [35]. Recently, detection of the DM with a  $Z$  boson at the LHC [36, 37] has been studied. In this paper, we investigate the DM pair production associated with a  $Z$  boson at the ILC.

The paper is arranged as follows: In Section II we briefly describe the related effective field theory and present the calculation strategy. In Section III, we present some numerical results and discussion. Finally, a short summary is given in Section IV.

## II. EFFECTIVE FIELD THEORY AND CALCULATION

The interactions between the SM and DM sectors are presumably effected by the exchange of some heavy mediators whose nature we do not need to specify, but only assume that they are much heavier than the typical scales. The interactions between the DM and SM leptons are described by an effective Lagrangian as

$$\mathcal{L} = \sum_{\ell} \left\{ \frac{1}{\Lambda_{D5}^2} \bar{\ell} \gamma^{\mu} \ell \bar{\chi} \gamma_{\mu} \chi + \frac{1}{\Lambda_{D8}^2} \bar{\ell} \gamma^{\mu} \gamma^5 \ell \bar{\chi} \gamma_{\mu} \gamma^5 \chi + \frac{1}{\Lambda_{D9}^2} \bar{\ell} \sigma^{\mu\nu} \ell \bar{\chi} \sigma_{\mu\nu} \chi \right\}, \quad (1)$$

where  $\chi$  is the DM particle assumed to be a Dirac fermion,  $\ell$  represents a lepton, and the effective scales  $\Lambda_{D5}$ ,  $\Lambda_{D8}$  and  $\Lambda_{D9}$  parameterize the vector (D5), axial-vector (D8) and tensor (D9) interactions between the two sectors, respectively. We will typically consider one type of interaction to dominate at a time, and will thus keep one  $\Lambda$  scale finite while the rest are set to infinity and decoupled.

There are two Feynman diagrams contributing to the  $e^+e^- \rightarrow \chi\bar{\chi}Z$  process at the leading order (LO), shown in Fig.1. The amplitudes for the two diagrams are given by

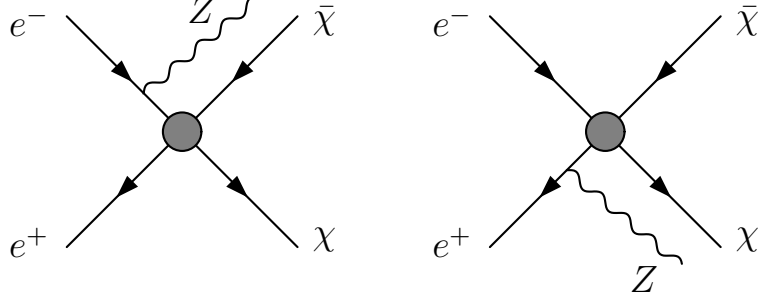


FIG. 1: Tree-level Feynman diagrams for the  $e^+e^- \rightarrow \chi\bar{\chi}Z$  process.

$$\begin{aligned}\mathcal{M}_{i1} &= \frac{1}{\Lambda_i^2} \bar{u}(p_4) \Gamma_i v(p_3) \bar{v}(p_2) \Gamma^i \frac{i}{\not{p}_1 - \not{p}_5 - m_e} \frac{ie\gamma^\mu}{4 \sin \theta_W \cos \theta_W} (1 - 4 \sin^2 \theta_W - \gamma^5) u(p_1) \epsilon_\mu^*(p_5), \\ \mathcal{M}_{i2} &= \frac{1}{\Lambda_i^2} \bar{u}(p_4) \Gamma_i v(p_3) \bar{v}(p_2) \frac{ie\gamma^\mu}{4 \sin \theta_W \cos \theta_W} (1 - 4 \sin^2 \theta_W - \gamma^5) \frac{i}{\not{p}_5 - \not{p}_2 - m_e} \Gamma_i u(p_1) \epsilon_\mu^*(p_5),\end{aligned}\quad (2)$$

where  $\Lambda_i = \Lambda_{D5}, \Lambda_{D8}, \Lambda_{D9}$  and  $\Gamma_i = \gamma_\mu, \gamma_\mu \gamma_5, \sigma_{\mu\nu}$  for vector, axial-vector and tensor interactions, respectively.  $p_i$  ( $i = 1, \dots, 5$ ) are the four-momenta of the incoming electron, positron and the outgoing dark matter pair and  $Z$  boson, separately. The differential cross section for the  $e^+e^- \rightarrow \chi\bar{\chi}Z$  process at tree-level is then obtained as

$$d\sigma_{tree} = \frac{1}{4} \frac{(2\pi)^4}{2s} \sum_{spin} |\mathcal{M}_{tree}|^2 d\Phi_3, \quad (3)$$

where  $\mathcal{M}_{tree} = \mathcal{M}_{i1} + \mathcal{M}_{i2}$  is the amplitude of all the tree-level diagrams shown in Fig.1. The factor  $\frac{1}{4}$  is due to taking average over the spins of the initial particles.  $d\Phi_3$  is the three-particle phase space element defined as

$$d\Phi_3 = \delta^{(4)} \left( p_1 + p_2 - \sum_{i=3}^5 p_i \right) \prod_{j=3}^5 \frac{d^3 \vec{p}_j}{(2\pi)^3 2E_j}. \quad (4)$$

In our calculations we adopt the 't Hooft-Feynman gauge. The FeynArts3.4 package [38] is used to generate the Feynman diagrams and convert them into the corresponding amplitudes. The amplitude reductions are mainly implemented by employing FormCalc5.4 package [39].

### III. NUMERICAL RESULTS AND DISCUSSION

In this section we present the numerical results for the  $e^+e^- \rightarrow \chi\bar{\chi}Z$  process at the ILC. The input parameters are taken as [40]

$$\alpha^{-1} = 137.036, \quad m_Z = 91.1876 \text{ GeV}, \quad m_W = 80.385 \text{ GeV}, \quad m_e = 0.511 \text{ MeV}. \quad (5)$$

By using the masses of  $W$  and  $Z$  bosons, we can obtain the value of weak mixing angle  $\sin^2 \theta_W = 1 - \frac{m_W^2}{m_Z^2} = 0.222897$ .

For the  $e^+e^- \rightarrow \chi\bar{\chi}Z$  process, the final produced particles  $\chi$  and  $\bar{\chi}$  are the missing energy which will escape the detector without being detected, and the  $Z$  boson can be efficiently identified by its leptonic decay. The SM background mainly comes from the  $e^+e^- \rightarrow \nu_\ell\bar{\nu}_\ell Z$  ( $\ell = e, \mu, \tau$ ) processes, where the neutrino is also the missing energy. Bhabha scattering of leptons with an additional  $Z$  boson,  $e^+e^- \rightarrow e^+e^-Z$ , is an important background, which has a large cross section but a very small selection efficiency, since both final state leptons must be undetected. As a simple analysis, we don't consider the background contribution of this part. There are two other important backgrounds, which are  $e^+e^- \rightarrow W^+W^-$  and  $W^\pm l^\mp \nu$  production when the  $W$  boson(s) decays leptonically. By adopting appropriate event selection, these backgrounds can also be safely ignored [41].

In Fig.2, we present the cross sections as functions of the colliding energy  $\sqrt{s}$  for the signal induced by the vector, axial-vector, tensor operators and the background including three generations of neutrinos by taking  $m_\chi = 10$  GeV and  $\Lambda = 1$  TeV with  $10^\circ < \theta_z < 170^\circ$  (There  $\theta_z$  is the angle between the  $Z$  boson and the incoming electron beam.), separately. From this figure we can see that, with the increment of the colliding energy  $\sqrt{s}$ , the cross sections for the signal process  $e^+e^- \rightarrow \chi\bar{\chi}Z$  induced by the vector, axial-vector and tensor operators increase rapidly, the cross section for the background from  $\nu_e\bar{\nu}_e Z$  production vary slowly, while those backgrounds from  $\nu_\mu\bar{\nu}_\mu Z$  and  $\nu_\tau\bar{\nu}_\tau Z$  production processes decrease slightly. With the increment of  $\sqrt{s}$ , the ratio of background to signal declines gradually and the signal becomes significant relative to the background. Consequently, we can obtain larger cross section for the signal and improve the probability for searching DM by raising the colliding energy  $\sqrt{s}$ .

In Fig.3, we present the DM mass dependence of the cross sections for the  $e^+e^- \rightarrow \chi\bar{\chi}Z$  process induced by the vector, axial-vector and tensor operators by taking  $\sqrt{s} = 500$  GeV,  $\Lambda = 1$  TeV and  $10^\circ < \theta_z < 170^\circ$ , separately. As shown in this figure, the cross section is insensitive to the DM mass  $m_\chi$  in the range of  $m_\chi < 100$  GeV, and decreases rapidly with the increment of  $m_\chi$  when  $m_\chi > 100$  GeV, due to the rapidly reduced phase space. We also conclude that the contributions from the spin-independent operator (D5) and spin-dependent operator (D8) can be distinguished until  $m_\chi > 100$  GeV.

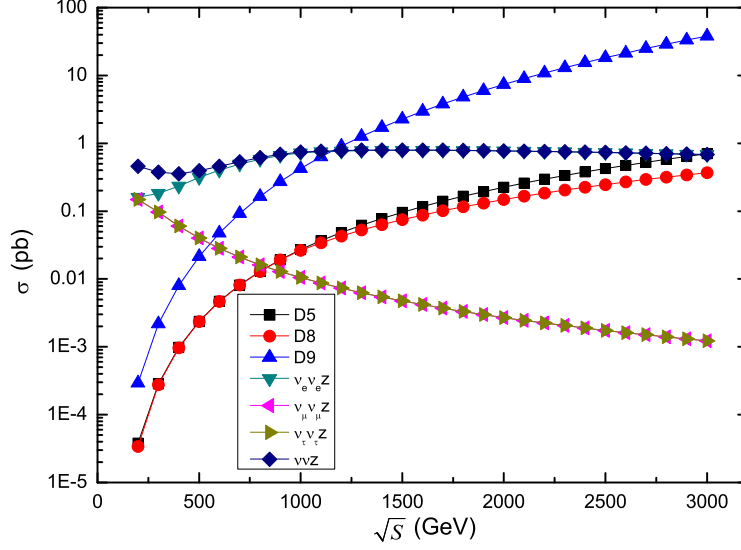


FIG. 2: Total cross sections for the signal process  $e^+e^- \rightarrow \chi\bar{\chi}Z$  induced by the vector (D5), axial-vector (D8), tensor (D9) operators and the background processes  $e^+e^- \rightarrow \nu_\ell \bar{\nu}_\ell Z$  ( $\ell = e, \mu, \tau$ ) as functions of the colliding energy  $\sqrt{s}$  by taking  $m_\chi = 10$  GeV,  $\Lambda = 1$  TeV and  $10^\circ < \theta_z < 170^\circ$ .

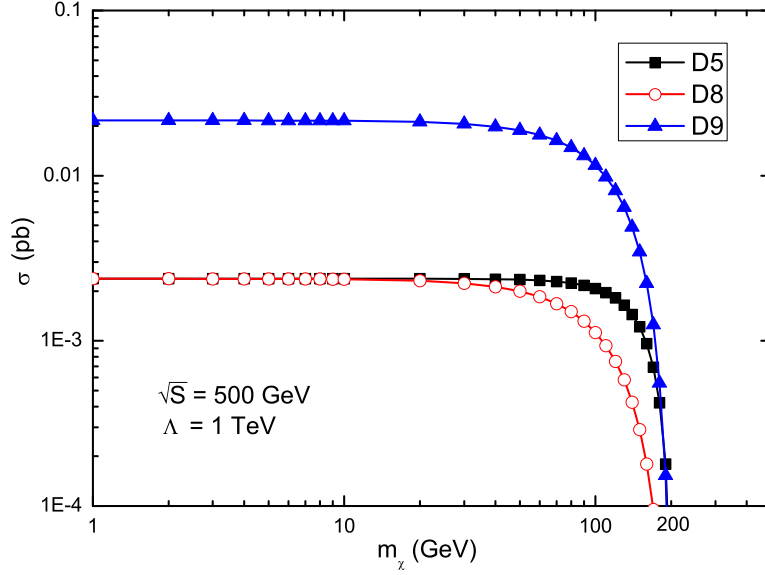


FIG. 3: Dependence of cross sections for the signal process  $e^+e^- \rightarrow \chi\bar{\chi}Z$  induced by the vector (D5), axial-vector (D8) and tensor (D9) operators on the DM mass with  $\sqrt{s} = 500$  GeV,  $\Lambda = 1$  TeV and  $10^\circ < \theta_z < 170^\circ$ .

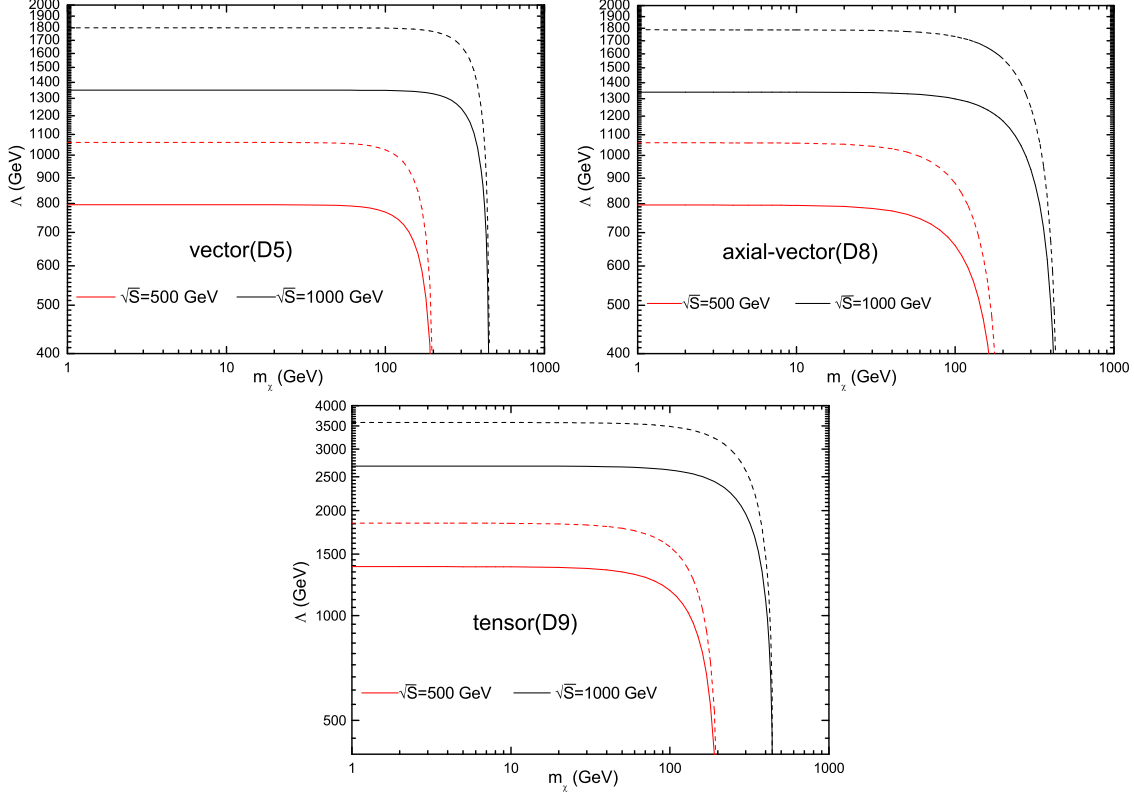


FIG. 4:  $3\sigma$  detection region on the  $m_\chi - \Lambda$  plane for the  $\chi\bar{\chi}Z$  production induced by the vector (D5), axial-vector (D8) and tensor (D9) operators at the  $\sqrt{s} = 500$  and  $1000$  GeV ILC with integrated luminosities of  $100fb^{-1}$  (solid lines) and  $1000fb^{-1}$  (dashed lines), respectively.

The significance of signal over background  $S$  is defined as

$$S = \frac{N_S}{\sqrt{N_B}} = \frac{\sigma_S \sqrt{\mathcal{L}}}{\sqrt{\sigma_B}}, \quad (6)$$

where  $N_{S,B}$  and  $\sigma_{S,B}$  are the event numbers and cross sections for signal and background, and  $\mathcal{L}$  denotes the integrated luminosity. In Fig.4, we depict the  $3\sigma$  detection region (defined as  $S \geq 3$ ) on the  $m_\chi - \Lambda$  plane by taking  $\sqrt{s} = 500$  and  $1000$  GeV,  $\mathcal{L} = 100$  and  $1000fb^{-1}$  and  $10^\circ < \theta_z < 170^\circ$ , respectively. As mentioned in Fig.2, we know that  $\Lambda$  has a larger space of adjustment by improving colliding energy. Of course, we can also reach the same effectiveness with the improvement of integrated luminosity. As shown Fig.4, we can see that  $\Lambda$  will increase with the improvement of  $\sqrt{s}$  or integrated luminosity. In Table I, we list the cross sections for the signal process  $e^+e^- \rightarrow \chi\bar{\chi}Z$  and the SM background at the  $\sqrt{s} = 500, 1000$  GeV ILC, and the corresponding significances with  $\mathcal{L} = 100$  and  $1000fb^{-1}$ , where the DM mass  $m_\chi$  and the energy scale  $\Lambda$  are taken as  $10$  GeV and  $1$  TeV, respectively.

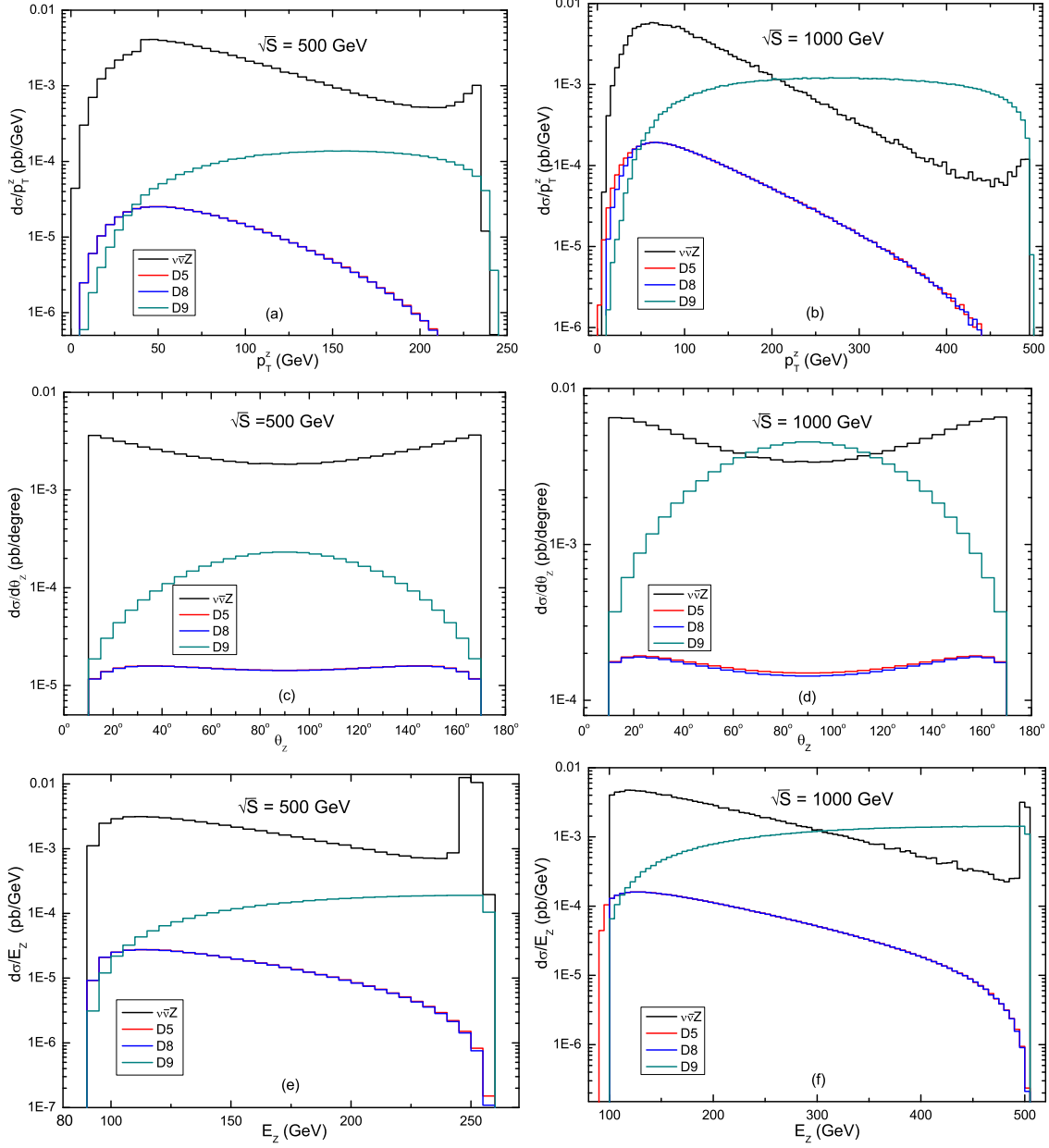


FIG. 5: Differential distributions of  $p_T^Z$ ,  $\theta_Z$  and  $E_Z$  for the signal induced by the vector, axial-vector, tensor operators and the background at the  $\sqrt{s} = 500$  and  $1000$  GeV ILC with  $m_\chi = 10$  GeV,  $\Lambda = 1$  TeV and  $10^\circ < \theta_z < 170^\circ$ .

Since the cross section for the signal is proportional to  $1/\Lambda^4$ , we can transform the function of cross section into a limit on the parameter  $\Lambda$ .

In Fig.5 we present the transverse momentum ( $p_T^Z$ ), angle ( $\theta_Z$ ) and energy ( $E_Z$ ) distributions of the final visible  $Z$  boson for the signal induced by the vector, axial-vector,



TABLE I: Cross sections for the signal process  $e^+e^- \rightarrow \chi\bar{\chi}Z$  and the SM background, and the corresponding significances, at the  $\sqrt{s} = 500, 1000$  GeV ILC with  $m_\chi = 10$  GeV,  $\Lambda = 1$  TeV,  $10^\circ < \theta_z < 170^\circ$  and two typical luminosity values of  $\mathcal{L}_1 = 100 \text{ fb}^{-1}$  and  $\mathcal{L}_2 = 1000 \text{ fb}^{-1}$ .

$\sqrt{s}$ (TeV)		$\sigma_S$ (fb)	$\sigma_B$ (fb)	$\sigma_S\sqrt{\mathcal{L}_1}/\sqrt{\sigma_B}$	$\sigma_S\sqrt{\mathcal{L}_2}/\sqrt{\sigma_B}$
0.5	D5	2.38		1.20	3.80
	D8	2.36	392.31	1.19	3.77
	D9	21.48		10.84	34.29
1.0	D5	27.13		9.96	31.51
	D8	26.35	741.42	9.68	30.60
	D9	423.18		155.42	491.47

tensor operators and background at the  $\sqrt{s} = 500$  and  $1000$  GeV ILC, respectively, with  $m_\chi = 10$  GeV,  $\Lambda = 1$  TeV and  $10^\circ < \theta_z < 170^\circ$ . The black, red, blue and green curves are for the SM background and the signal induced by vector, axial-vector and tensor operators, respectively. Our results show that the background is very large compared to the signal. In order to efficiently separate the  $\chi\bar{\chi}Z$  signal from the  $\nu\bar{\nu}Z$  background, we need to adopt a proper event selection to increase the signal-to-background ratio. For the tensor(D9) interaction, the differential distribution of the signal is very different from the background, and the differential cross section is much larger than the contributions from D5 and D8. By taking appropriate cuts, it is easily to separate the signal from the background, and raise the limit on parameter  $\Lambda_{D9}$ . However, the differential cross sections of the D5 and D8 are small relative to that of D9, and the change of the signal distribution is consistent with the background. It is difficult to distinguish the signal from the background through the cuts of  $p_T^Z$ ,  $\theta_Z$ ,  $E_Z$ .

#### IV. SUMMARY

The origin of dark matter remains one of the most compelling mysteries in our understanding of the universe today. High energy colliders are ideal facilities to search for DM. In this paper, we study the effects of the effective operators of DM via dark matter pair production associated a  $Z$  boson at the ILC. The SM background  $e^+e^- \rightarrow \nu_\ell\bar{\nu}_\ell Z$  ( $\ell = e, \mu, \tau$ )

is also considered for comparison. We obtain the cross sections as functions of colliding energy  $\sqrt{s}$  and the DM mass  $m_\chi$  for the signal induced by the vector, axial-vector, tensor operators and the SM background. We find that raising the colliding energy can improve the probability for searching DM, and the contributions from the spin-independent operator (D5) and the spin-dependent operator (D8) can be distinguished until  $m_\chi > 100$  GeV. If this signal is not observed at the ILC, we set a lower limit on the new physics scale  $\Lambda$  at the  $3\sigma$  level. Meanwhile, we show the differential distributions of  $p_T^Z$ ,  $\theta_Z$  and  $E_Z$ . We find that it is easy to separate the signal from the background by taking appropriate cuts for the tensor (D9) interaction, but difficult for the D5 and D8 interactions. We conclude that the ILC has the potential to detect the  $e^+e^- \rightarrow \chi\bar{\chi}Z$  production.

## V. ACKNOWLEDGMENTS

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